Exp5：Rotational Inertia and Conservation of Angular Momentum

Measured the rotational inertia of different shape of objects. [2]

[A] Rotational Inertia of a Point Mass

[B] Rotational Inertia of Disk and Ring

[C] Rotational Inertia of Disk Off-Axis

[D] Conservation of Angular Moment

I. Theory [1]

When the object moving speed is lower than speed of light, the motion of object can describe by Newton's Second Law.

$$F = ma \qquad (1)$$

M is inertia mass, F is the net force on the body, a is acceleration of motion. For rotation motion, the eq. of motion describe by net torque (τ)、moment of inertia (I) and angular acceleration (α)：

$$\tau = I\,\alpha \qquad (2)$$

angular velocity (ω = rotating radius (r) × tangential acceleration (a_t)). Table 1 is some corresponding relations for translational and rotational motion.

Table 1 Some Corresponding Relations for Translational and Rotational Motion.

Pure Translation (x axis)		Pure Rotation (Symmetry about a Fixed Rotation Axis)	
Position component	x	Rotational position component	θ
Velocity component	$v_x = dx/dt$	Rotational velocity component	$\omega = d\theta/dt$
Acceleration	$a_x = dv_x/dt$	Rotational acceleration component	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's Second Law	$F_x^{net} = ma_x$	Newton's Second Law	$\tau^{net} = I\alpha$
Work	$W = \int F_x\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv_x^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power	$P = F_x v_x$	Power	$P = \tau\omega$
Work-kinetic energy theorem	$W = \Delta K$	Work-kinetic energy theorem	$W^{rot} = \Delta K^{rot}$

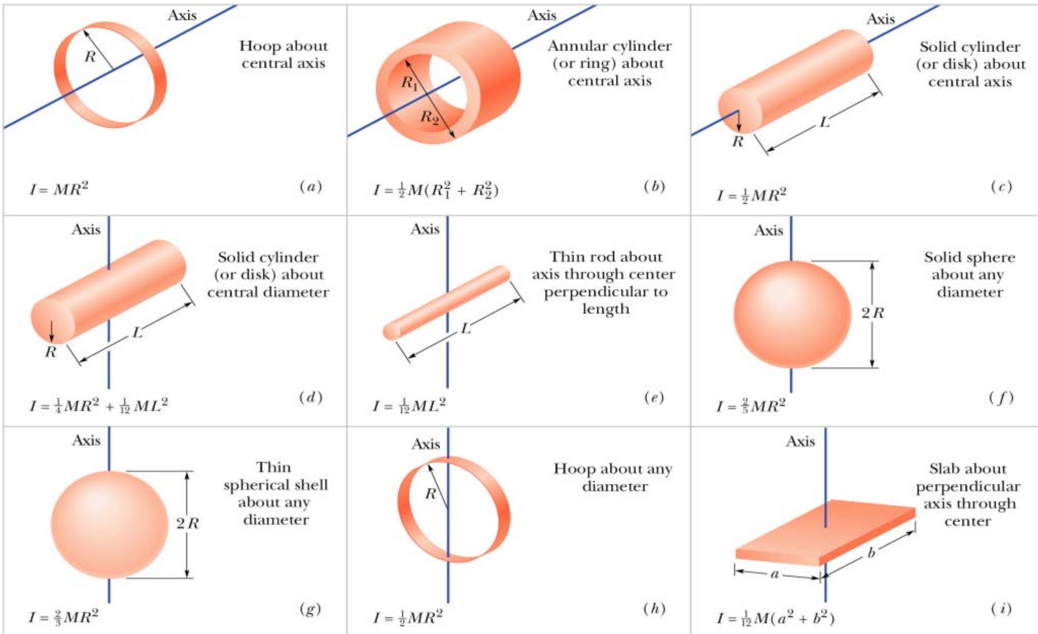
(1) Rotational Inertia

When a force F causes a rigid body of mass m to accelerate along a coordinate axis, the force does work W on the body. Now consider a rational situation that is similar. The body's rotational kinetic energy (K= (1/2) I ω^2) can change. I is the rotational inertia of the body about the fix axis and w is the angular speed of body. That quantity the rotational inertia I of the body with respect to axis of rotation. It is constant for a particular rigid body and a particular rotation axis.

$I=\Sigma m_i r_i$ (rotational inertia)

Table 2 gives the results of such integration for nine common body shapes and the indicated axis of rotation.

Table 2 Some Rotational inertias



II. Equipment：As Shown in Figure 1

Before Exp:

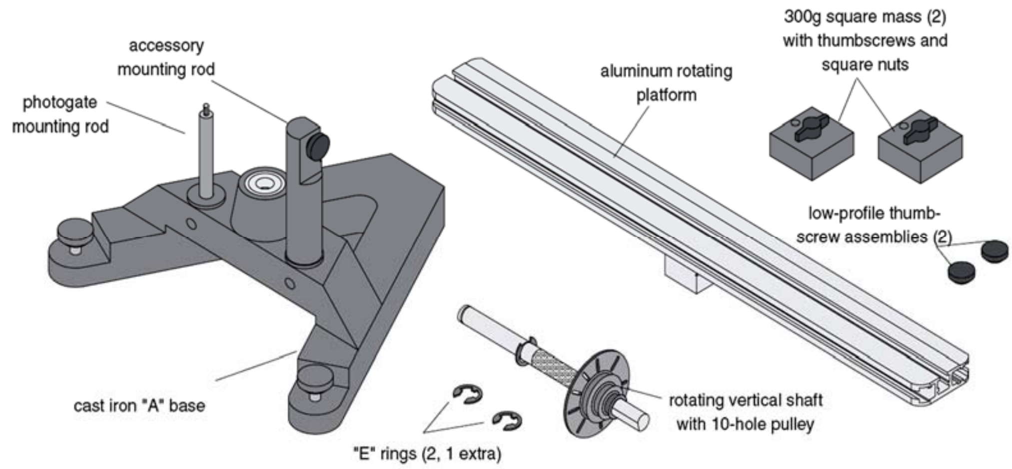
(1) Arduino (open-source electronics platform) **Software download:**

Arduino Introduction: https://www.arduino.cc/

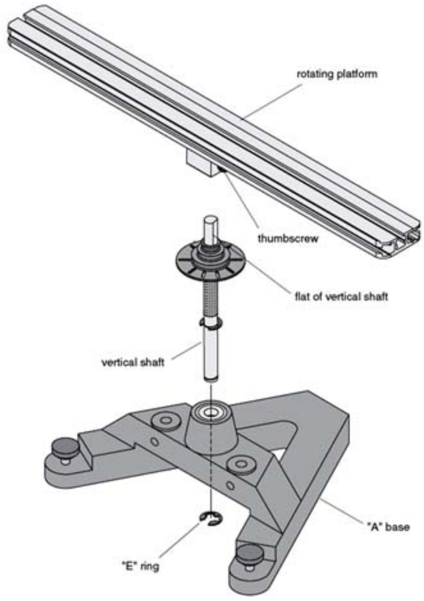
(2) **CoolTerm**: http://freeware.the-meiers.org

Used Coolterm to get database and load out to EXCEL for graphing.

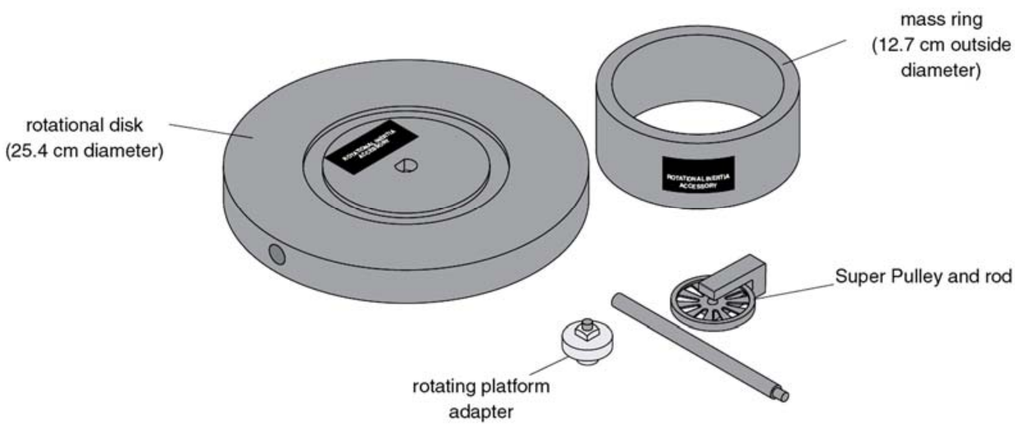
1. **Rotating platform :** As shown in figure 1(a). . Include "A" base, rotating parts and rotating platform.
2. **Accessory of rotation inertia**: As shown in figure 1(c). Include rotational disk (25.4 cm diameter), mass ring (12.7 cm outside diameter), 10-spoke pulley and rotating platform adapter.
3. **Photogate**: As shown in figure 1(d)
4. **Precision Timer Program** (Arduino box01)



(a) Rotating Platform Equipment



(b) Rotating Platform Setup



(c) Rotational rigid body and pulley component

(d) Photogate **(f) Arduino box01 (data receiver)**

- Figure 1 Rotating platform. (a) Accessory, (b) Setup, (c) rotational disk, mass ring and pulley, (d) photogate, (f) Precision Timer Program (Arduino box)

III. Equipment introduction and setup

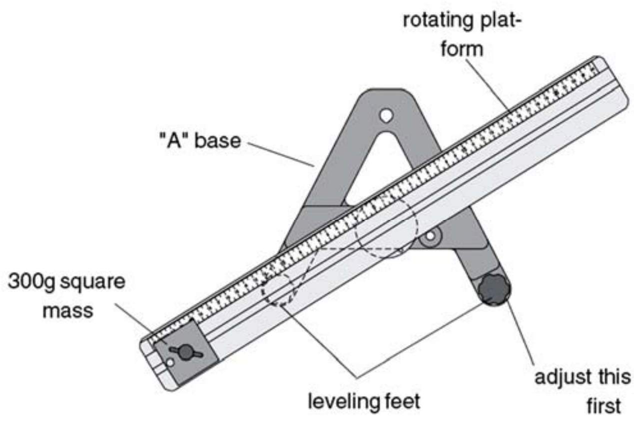
1. configuration of rotation platform：

As shown in figure 1(a) and figure 1(b).

(1) Insert the cylindrical end of the shaft into the bearings on the top-side of the “A” base. Secure the shaft in place by inserting the “E” ring in the slot at the bottom of the shaft.

(2) 。Mount the track to the shaft and tighten the thumb screw against the flat side of the “D” on the shaft. As shown in Figure 1(b).

2. Leveling the Base： Some experiments require the apparatus to be extremely level. If the track is not level, the uneven performance will affect the results. To level the base, perform the following steps:



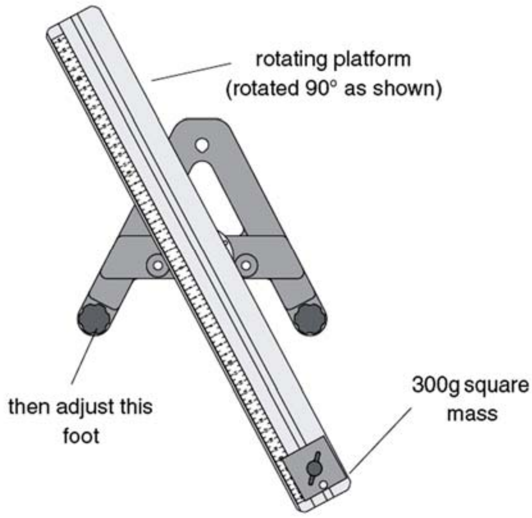


Figure 2 (a) (b)

Figure 2 Leveling the Base

(1) As shown in Figure 2. Purposely make the platform unbalanced by attaching the 300 g square mass onto either end of the aluminum track. Tighten the screw so the mass will not slide. If the

hooked mass is hanging from the side post in the centripetal force accessory, place the square mass on the same side.

(2) As shown in Figure 2(a). Adjust the leveling screw on one of the legs of the base until the end of the track with the square mass is aligned over the leveling screw on the other leg of the base.

(3) Rotate the track 90 degrees so it is parallel to one side of the "A" base and adjust the other leveling screw until the track will stay in this position.

(4) The track is now level and it should remain at rest regardless of its orientation.

3. Installing the Optional Smart Pulley Photogate Head, as shown in figure 3.

(1) To use the Photogate Only

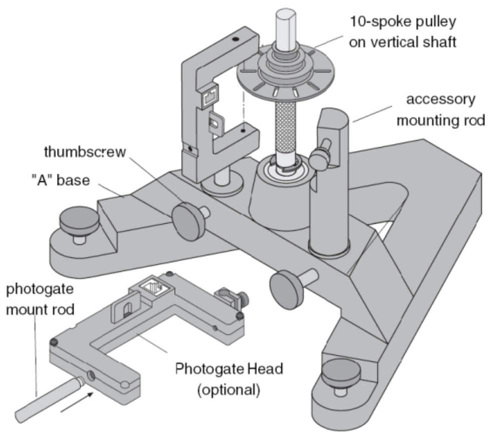


Figure 3a

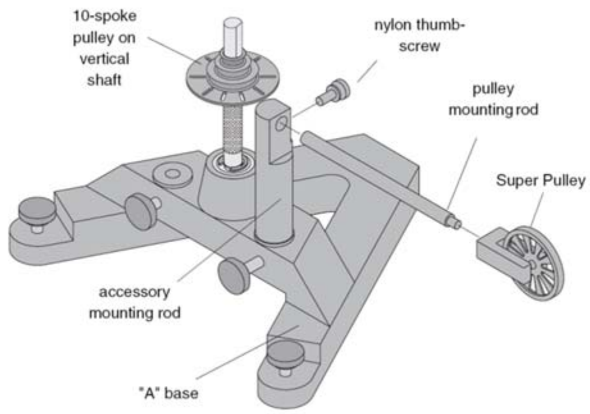


Figure 3b

Figure 3 Using the Accessory Mounting Rod With the Pulley

(1) To install the black rod to the base by inserting the rod into either hole adjacent to the center shaft on the base.

(2) As shown in 3a. Loosen the thumb screw on the base to allow the black rod to rotate. Orient the rod and photogate head so the infrared beam passes through the holes in the pulley. If the photogate head is powered by a computer, you can tell when the photogate is blocked by watching the LED indicator on the end of the photogate. The photogate head should not be rubbing against the pulley. When the head is in the correct position, tighten the bottom screw to fix the rod in place.

4. To use the Super Pulley and the Pulley Mounting Rod:

(1) As shown in 3b. Insert the Smart Pulley rod into the hole in the black rod and tighten the set screw against the Smart Pulley rod.

(2) Adjust the position of the base so the string passing over the Smart Pulley will clear the edge of the table.

5. Rotational Inertia Accessory Assembly

Little assembly is required to use the Rotational Inertia Accessory. The rotational disk can be placed directly onto the axle of the rotating base or can be used with the rotating platform via the included platform adapter.

Platform Adapter Assembly

1. Attach the square nut (supplied with the Rotational Inertia Accessory) to the platform adapter.
2. Position the platform adapter at the desired radius as shown in Figure 4.
3. Grip the knurled edge of the platform adapter and tighten

The rotating disk can be mounted in a variety of positions using any of the four holes on the rotation disk.

• Two “D” holes exist on the edge of the disk, located at 180° from one another.

• One “D” hole is located at the center on the top surface (the surface with the metal ring channel and the label) of the disk.

• One hole is located at the center on the bottom surface of the disk and is actually the inner race of a bearing. This enables the rotational disk to rotate (in either direction) in addition to other rotating motions applied to your experiment setup.

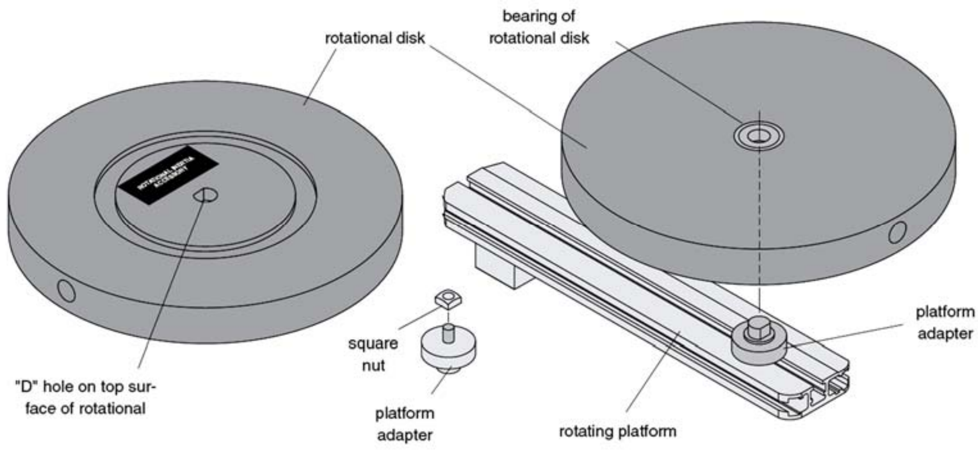


Fig 4 Rotational Inertia Accessory Including Platform Adapter Assembly

[A] Rotational Inertia of a Point Mass

I. Purpose:

The purpose of this experiment is to find the rotational inertia of a point mass experimentally and to verify that this value corresponds to the calculated theoretical value.

II. Theory

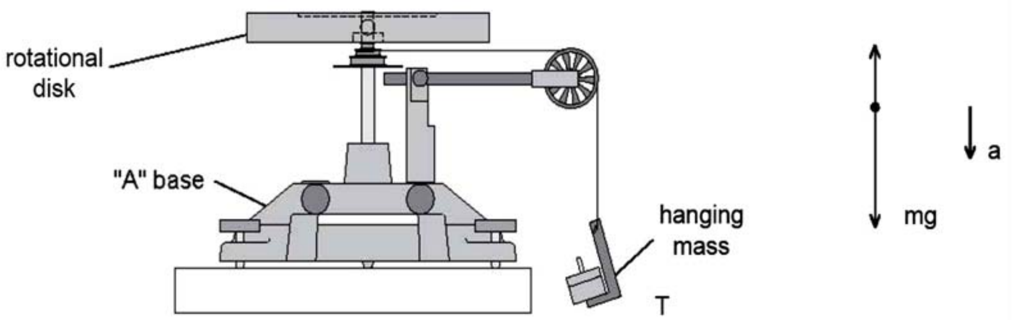
Theoretically, the rotational inertia, I, of a point mass is given by $I = MR^2$, where M is the mass, R is the distance the mass is from the axis of rotation.

To find the rotational inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Since $\tau = I\alpha$,

where α is the angular acceleration which is equal to a/r and τ is the torque caused by the weight hanging from the thread which is wrapped around the step pulley below the rotating platform, and

$$\vec{\tau} = \vec{r} \times \vec{T}$$

where r is the radius of the step pulley about which the thread is wound and T is the tension in the thread when the platform is rotating.



Applying Newton' s Second Law for the hanging mass, m, gives

$$\Sigma F = mg - T = ma \; ; \; T = m(g-a)$$

Solving for the tension in the thread gives

$$\tau = rT = mr\left(g - a\right)$$

Once the linear acceleration of the mass (m) is determined, the torque and the angular acceleration can be obtained for the calculation of the rotational inertia.

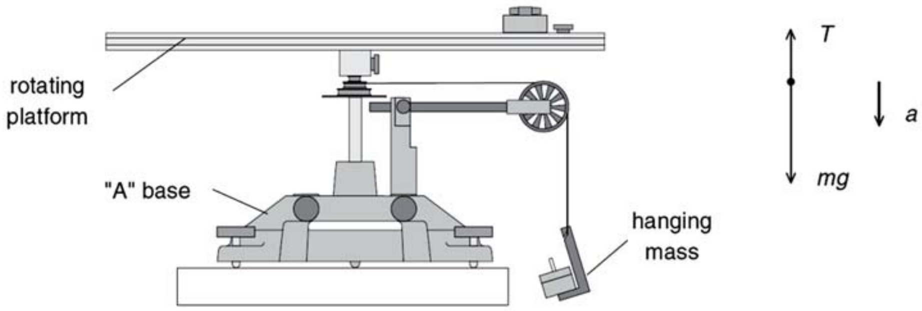


Figure 5 Rotational Inertia Experimental Setup

III、Equipment

- Precision Timer Program (Arduino box)
- Mass and Hanger Set
- Caliper
- 10-spoke pulley
- Photogate
- Balance

IV. Experiment Setup

1. Level the rotating platform.
2. Attach the square mass (point mass) to the track on the rotating platform at any radius you wish.
3. Mount the Photogate/Pulley system to the base and connect the photogate through an interface (Arduino box) to a computer. See Figure 6.
4. Attach a thread to the middle step of the step pulley and hang the thread over the 10-spoke pulley.

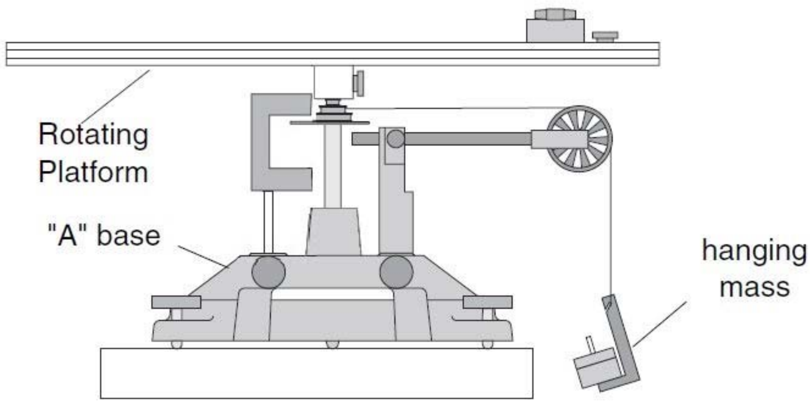


Figure 6 Rotational inertia of a point mass

Procedure

1. Weigh the square mass to find the mass M and measure the distance R from the axis of

rotation to the center of the square.

Table 1.1 Theoretical Rotational Inertia

Mass M (kg)	Radius R (m)	Rotational Inertia I	
		Theoretical Value	Experimental Value

2. **Accounting for Friction:** Hang a small amount of mass - such as a few paper clips - on the end of the thread that is over the pulley. Adjust the amount of mass on the thread by adding or removing a paper clip. Until the velocity stays constant as the mass falls.
3. **Finding the Acceleration of the Point Mass and Equipment**

To find the acceleration, put about 50 g - measure the exact mass and record it in Table 1.2 - on the end of the thread over the pulley.

1. Wind the thread up and hold the Rotating Platform.
2. Let the platform begin to turn and at the same time, start recording data.
3. Let the mass fall toward the floor but STOP recording data just before the mass hits the floor.
4. Examine your Graph display of Velocity versus Time. The slope of the best 'Linear Fit' for your data is the acceleration of the platform.
5. Record the slope in Table 1.2.

Table 1.2 Rotational Inertia Data

	Point Mass and platform	platform Alone
Friction Mass		
Hanging Mass		
Slope		
Radius		

Calculations

Calculate the theoretical value of the rotational inertia of the point mass. Record in Table 1.3.

Table 1.3 Results

Rotational Inertia for Point Mass and platform Combined	
Rotational Inertia for platform Alone	
Rotational Inertia for Point Mass (experimental value)	

Rotational Inertia for Point Mass (theoretical value)	
% Difference	

V. Question:

Why need to put a square mass on the platform when leveling?

[B] Rotational Inertia of Disk and Ring

I. Purpose

The purpose of this experiment is to find the rotational inertia of a ring and a disk experimentally and to verify that these values correspond to the calculated theoretical values.

II. Theory

Theoretically, the rotational inertia, I, of a ring about its center of mass is given by:

$$I = \frac{1}{2} M \left(R_1^2 + R_2^2 \right)$$

where M is the mass of the ring, R_1 is the inner radius of the ring, and R_2 is the outer radius of the ring. See Figure 7.

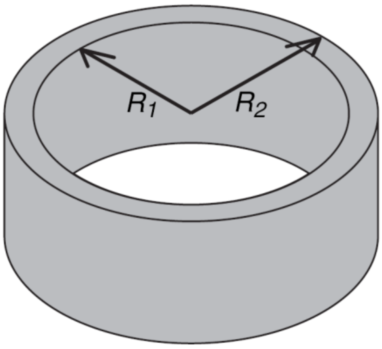


Figure 7 Ring

The rotational inertia of a disk about its center of mass is given by:

$$I = \frac{1}{2} MR^2 \quad R \text{ the radius of disk}$$

To find the rotational inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Since $\tau = I\alpha$, where α is the angular acceleration which is equal to a/r and τ is the torque caused by the weight hanging from the thread which is wrapped around the base of the ring.

$$\vec{\tau} = \vec{r} \times \vec{T}$$

where r is the radius of the cylinder about which the thread is wound and T is the tension in the thread when the ring is rotating.

Applying Newton's Second Law for the hanging mass, m, gives

$$\Sigma F = mg - T = ma$$

Solving for the tension in the thread gives:

$$T = m\left(g - a\right)$$

Once the linear acceleration of the mass (m) is determined, the torque and the angular acceleration can be obtained for the calculation of the rotational inertia.

III. Equipment

- Rotational Inertia Accessory

- Precision Time Program (Arduino box)
- Mass and Hanger Set
- 10-spoke pulley
- Calipers
- Balance

IV. Experiment Setup

1. Remove the track from the Rotating Platform and place the disk directly on the center shaft as shown in Figure 8. The side of the disk that has the indentation for the ring should be up.
2. Place the ring on the disk, seating it in this indentation.
3. Mount the Photogate/Pulley System to the base and connect it to computer

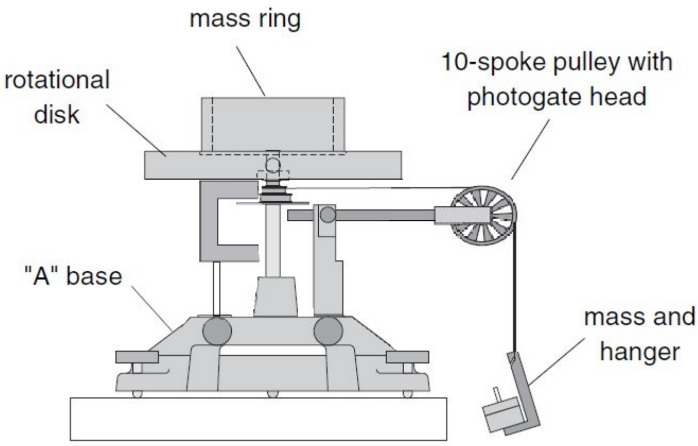


Figure 8

4. Weigh the ring and disk to find their masses and record these masses in Table 3.1.
5. Measure the inside and outside diameters of the ring and calculate the radii R_1 and R_2. Record in Table 3.1.

Table 3.1 Theoretical Value Rotational Inertia

	Mass	Radius	
Ring		Inner	Outer
Disk			
Theoretical Value Rotational Inertia			

6. Measure the mass on the end of the thread and record it as the “Friction Mass” in Table 3.2.

Table 3.2

	Ring and Disk Combined	Disk Alone	Disk Vertical

Friction Mass			
Hanging Mass			
Slope			
Radius			

Finding the Acceleration of Ring and Disk

To find the acceleration, put about 50 g - record the exact hanging mass in Table 3.2 - over the pulley.

1. Wind the thread up and hold the Rotating Platform.
2. Let the Rotating Platform begin to turn and at the same time, start recording data.
3. Let the mass descend toward the floor but STOP recording data just before the mass hits the floor.
4. Examine your Graph display of Velocity versus Time. The slope of the best fit line for your data is the acceleration of the apparatus.
5. Record the slope in Table 3.2.

Measure the Radius

1. Using calipers, measure the diameter of the cylinder about which the thread is wrapped and calculate the radius. Record in Table 3.2.

Finding the Acceleration of the Disk Alone

Since in **Finding the Acceleration of Ring and Disk** the disk is rotating as well as the ring, it is necessary to determine the acceleration, and the rotational inertia, of the disk by itself so this rotational inertia can be subtracted from the total, leaving only the rotational inertia of the ring.

1. To do this, take the ring off the rotational apparatus and repeat **Finding the Acceleration of Ring and Disk** for the disk alone.

> **NOTE:** that it will take less "friction mass" to overcome the new kinetic friction and it is only necessary to put about 30 g over the pulley in **Finding the Acceleration of the Disk Alone**.

Disk Rotating on an Axis Through Its Diameter

Remove the disk from the shaft and rotate it up on its side. Mount the disk vertically by inserting the shaft in one of the two "D"-shaped holes on the edge of the disk.

> **WARNING!** Never mount the disk vertically using the adapter on the track. The adapter is too short for this purpose and the disk might fall over while being rotated.

Repeat steps **Measure the Radius** and **Finding the Acceleration of the Disk Alone** to determine the rotational inertia of the disk about its diameter. Record the data in Table 3.2.

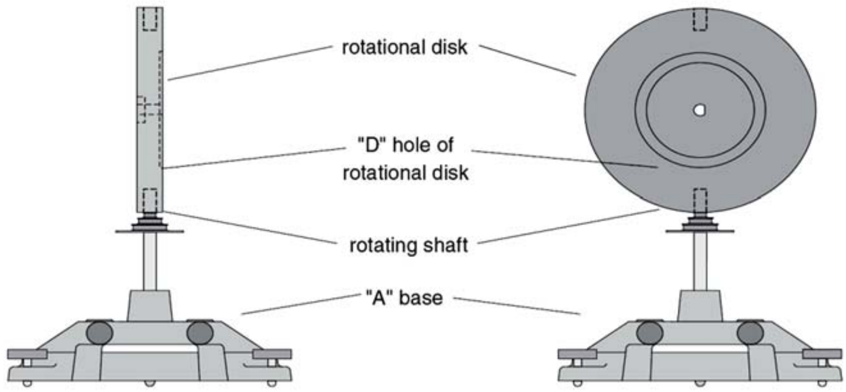


Figure 9

Calculations

Record the results of the following calculations in Table 3.3.

1. Subtract the “friction mass” from the hanging mass used to accelerate the disk to determine the mass, *m*, to be used in the equations.
2. Calculate the experimental value of the rotational inertia of the ring and disk together.
3. Calculate the experimental value of the rotational inertia of the disk alone.
4. Subtract the rotational inertia of the disk from the total rotational inertia of the ring and disk. This will be the rotational inertia of the ring alone.
5. Calculate the experimental value of the rotational inertia of the disk about its diameter.
6. Calculate the theoretical value of the rotational inertia of the ring.
7. Calculate the theoretical value of the rotational inertia of the disk about its center of mass and about its diameter.
8. Use a percent difference to compare the experimental values to the theoretical values.

Table 3.3 Result

Rotational Inertia for Ring and Disk			
	Rotational Inertia for Disk	Rotational Inertia for Ring	Rotational Inertia for Vertical Disk
Experimental Value			
Theoretical Value			
% Difference			

[C] Rotational Inertia of Disk Off-Axis

I. Purpose

The purpose of this experiment is to find the rotational inertia of a disk about an axis parallel to the center of mass axis.

II. Theory

Theoretically, the rotational inertia, I, of a disk about a perpendicular axis through its center of mass is given by:

$$I_{cm} = \frac{1}{2} MR^2$$

where M is the mass of the disk and R is the radius of the disk. The rotational inertia of a disk about an axis parallel to the center of mass axis is given by:

$$I = I_{cm} + Md^2$$

where *d* is the distance between the two axes.

In one part of this experiment, the disk is mounted on its ball bearing side which allows the disk to freely rotate relative to the track. So as the track is rotated, the disk does not rotate relative to its center of mass. Since the disk is not rotating about its center of mass, it acts as a point mass rather than an extended object and its rotational inertia reduces from:

$$I = I_{cm} + Md^2 \quad \text{to} \quad I = Md^2$$

To find the rotational inertia experimentally, a known torque is applied to the object and the resulting angular acceleration is measured. Since $\tau = I\alpha$, where α is the angular acceleration which is equal to a/r and τ is the torque caused by the weight hanging from the thread which is wrapped around the base of the apparatus.

$$\vec{\tau} = \vec{r} \times \vec{T}$$

where r is the radius of the cylinder about which the thread is wound and T is the tension in the thread when the apparatus is rotating.

Applying Newton's Second Law for the hanging mass, *m*, gives

$$\Sigma F = mg - T = ma$$

Solving for the tension in the thread gives:

$$T = m(g - a)$$

Once the linear acceleration of the mass (*m*) is determined, the torque and the angular acceleration can be obtained for the calculation of the rotational inertia.

III. Equipment

Precision Timer Program (Arduino box)

Rotational Inertia Accessory

Photogate

Mass and Hanger Set
Smart Pulley
Calipers
Balance

IV. Experience Setup

1. Set up the Rotational Accessory as shown in Figure 10. Mount the disk with its bearing side up. Use the platform adapter to fasten the disk to the track at a large radius.

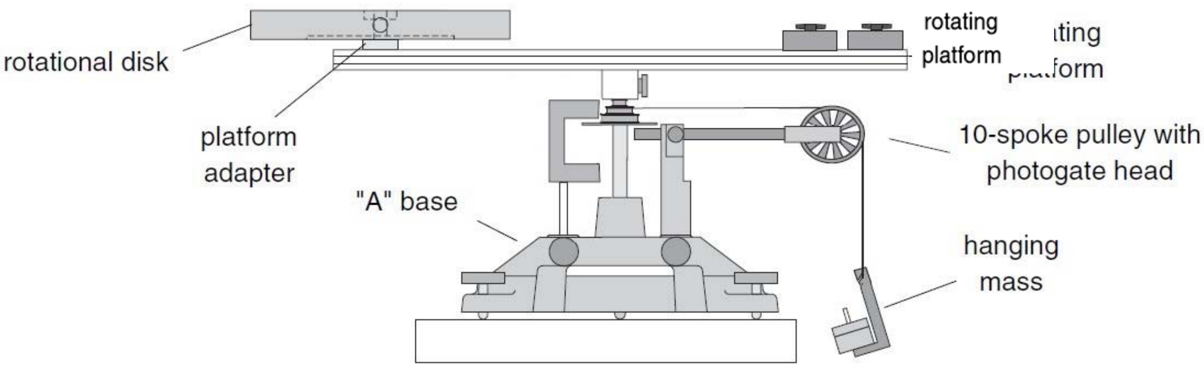


Figure 10

2. Mount the Smart Pulley to the base and connect it to a computer.
3. Weigh the disk to find the mass *M*. Measure the diameter and calculate the radius *R*. Measure the distance, *d*, from the axis of rotation to the center of the disk.

Table 4.1 Theoretical Rotational Inertia

Mass of Disk *M*	
Radius of Disk *R*	
Distance Between Parallel Axis *d*	

4. **Accounting for Friction:** Hang a small amount of mass (alone 5~15g) - such as a few paper clips - on the end of the thread that is over the pulley. Adjust the amount of mass on the thread by adding or removing a paper clip. Until the velocity stays constant as the mass falls.
5. To find the acceleration, put about 50 g - record the exact hanging mass in Table 4.2 - over the pulley.

Table 4.2

	Fixed Disk and Track Combined	Track Alone	Rotating Disk and Track Combined
Friction Mass			
Hanging Mass			

Slope			
Radius			

Measure the Radius

1. Using calipers, measure the diameter of the cylinder about which the thread is wrapped and calculate the radius. Record in Table 4.2.

Finding the Acceleration of Track Alone

Since in **Finding the Acceleration of Disk and Track** the track is rotating as well as the disk, it is necessary to determine the acceleration, and the rotational inertia, of the track by itself so this rotational inertia can be subtracted from the total, leaving only the rotational inertia of the disk.

1. To do this, take the disk off the rotational apparatus and repeat **Finding the Acceleration of Disk and Track** for the track alone.

> **NOTE:** It will take less "friction mass" to overcome the new kinetic friction and it is only necessary to put about 30 g over the pulley in **Finding the Acceleration of Track Alone**.

Disk Using Ball Bearings (Free Disk)

Mount the disk upside-down at the same radius as before. Now the ball bearings at the center of the disk will allow the disk to rotate relative to the track. Repeat **Accounting For Friction** and **Finding the Acceleration of Disk and Track** for this case and record the data in Table 4.2.

Calculations

Record the results of the following calculations in Table 4.3.

1. Subtract the "friction mass" from the hanging mass used to accelerate the apparatus to determine the mass, m, to be used in the equations.
2. Calculate the experimental value of the rotational inertia of the fixed disk and track combined.
3. Calculate the experimental value of the rotational inertia of the track alone.
4. Subtract the rotational inertia of the track from the rotational inertia of the fixed disk and track. This will be the rotational inertia of the fixed disk alone.
5. Calculate the experimental value of the rotational inertia of the fixed disk and track combined.
6. Subtract the rotational inertia of the track from the rotational inertia of the free disk and track. This will be the rotational inertia of the free disk alone.
7. Calculate the theoretical value of the rotational inertia of the fixed disk off axis.
8. Calculate the theoretical value of a point mass having the mass of the disk.

9. Use a percent difference to compare the experimental values to the theoretical values.

Table 4.3 Result

Fixed Disk and Track	
Free Disk and Track	
Track Alone	

	Fixed Disk Off-Axis	Free Disk Alone（Point Mass）
Theoretical Value		
Experimental Value		
% Difference		

[D] Conservation of Angular Moment

I. Purpose

A non-rotating ring is dropped onto a rotating disk and the final angular speed of the system is compared with the value predicted using conservation of angular momentum.

II. Theory

When the ring is dropped onto the rotating disk, there is no net torque on the system since the torque on the ring is equal and opposite to the torque on the disk. Therefore, there is no change in angular momentum. Angular momentum is conserved.

$$L = I_i \omega_i = I_f \omega_f$$

where I_i is the initial rotational inertia and ω_i is the initial angular speed. The initial rotational inertia is that of a disk :

$$\left(\frac{1}{2}\right) M_1 R^2$$

and the final rotational inertia of the combined disk and ring is

$$\mathrm{I}_f = \frac{1}{2} M_1 R^2 + \frac{1}{2} M_2 \left(r_1^2 + r_2^2\right)$$

So the final rotational speed is given by

$$\omega_f = \frac{M_1 R^2}{M_1 R^2 + M_2 \left(r_1^2 + r_2^2\right)} \omega_i$$

III. Equipment

- Smart Pulley Timer Program
- Photogate
- Rotational Inertia Accessory
- Rotating Platform
- Smart Pulley Photogate
- Balance

IV. Experience setup

1. Level the apparatus using the square mass on the track.
2. Mount the Photogate on the metal rod on the base and position it so it straddles the holes in the pulley on the center rotating shaft.
3. Assemble the Rotational Inertia Accessory as shown in Figure 11. The side of the disk with the indentation for the ring should be up.
4. Hold the ring just above the center of the disk.

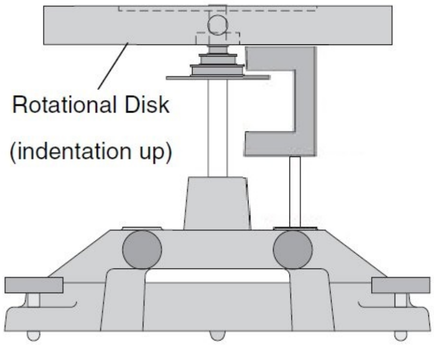


Figure 11

Procedure

1. Hold the ring just above the center of the disk. Give the disk a spin using your hand.
2. Start recording data. After about 25 data points have been taken, drop the ring onto the spinning disk. See Figure 12.
3. Continue to take data after the collision for a few seconds and then stop recording data.

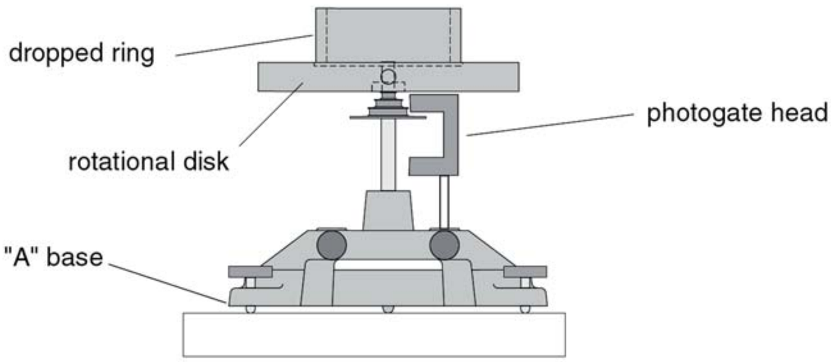


Figure 12

4.Graph display of Velocity (rad/s) versus Time (s). Determine the angular velocity immediately before and immediately after the collision. Record these values in Table 5.1.
5. Weigh the disk and ring and measure the radii. Record these values in Table 5.1.

Analysis

1. Calculate the expected (theoretical) value for the final angular velocity and record this value in Table 5.1.
2. Calculate the percent difference between the experimental and the theoretical values of the final angular velocity and record in Table 5.1.

Table 5.1

Initial Angular Speed：			
Final Angular Speed（experimental value）：			
	Mass	Radius	
Disk		Inner	Disk

Ring		
Final Angular Speed（theoretical value）		
% Difference Between Final Angular Speeds		

V. Question:

1. Does the experimental result for the angular speed agree with the theory?
2. What percentage of the rotational kinetic energy lost during the collision? Calculate this and record the results.

   Hint: $$\%KE\,Lost = \frac{\frac{1}{2}I_i\omega_i^2 - \frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_f\omega_f^2}$$

3. Compare the similarities and differences between part#B and part#E.

VI. Reference:

(1) Principles of Physics, Wiley, Tenth Edition, Rotational Motion

(2) ME-8950A Complete Rotational System Manual, PASCO Scientific Inc.